Possibility of Precise Measurement of the Cosmological Power Spectrum With a Dedicated 21cm Survey After Reionization

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(Dated: March 21, 2008)

Measurements of the 21cm line emission by residual cosmic hydrogen after reionization can be used to trace the power spectrum of density perturbations through a significant fraction of the observable volume of the Universe. We show that a dedicated 21cm observatory coule probe a number of independent modes that is two orders of magnitude larger than currently available, and enable a cosmic-variance limited detection of the signature of a neutrino mass ~ 0.05 eV. The evolution of the linear growth factor with redshift could also constrain exotic theories of gravity or dark energy to an unprecedented precision.

PACS numbers: 98.80-k, 95.30Dr, 95.55Jz

arXiv:0801.1677v2 [astro-ph] 21 Mar 2008

Recently, there has been much interest in the feasibility of mapping the three-dimensional (3D) distribution of cosmic hydrogen through its resonant spin-flip transition at a rest-frame wavelength of 21cm[1, 2]. Several experiments are currently being constructed (such as MWA[32], LOFAR[33], PAPER [34], 21CMA[35]) and more ambitious designs are being planned (SKA[36]) to detect the theoretically-predicted emission signal.

Measurements of the power-spectrum of 21cm brightness fluctuations could constrain the initial conditions from inflation as well as the nature of the dark matter and dark energy. The 21cm fluctuations are expected to simply trace the primordial power-spectrum of matter density perturbations either before the first galaxies had formed (at redshifts $z \gtrsim 20$)[3, 4] or after reionization ($1 \lesssim z \lesssim 6$) – when only dense pockets of self-shielded hydrogen (such as damped Lyα systems) survive [5, 6]. During the epoch of reionization, the fluctuations are mainly shaped by the topology of ionized regions [7, 8, 9], and thus depend on astrophysical details. However, even during this epoch, the line-of-sight anisotropy of the 21cm power spectrum due to peculiar velocities, can in principle be used to separate the implications for fundamental physics from the unknown details of the astrophysics [7, 10]. In what follows, we will focus our discussion on the post-reionization epoch [5, 6] which offers two advantages. First, it is least contaminated by the Galactic synchrotron foreground (whose brightness temperature scales with the redshift under consideration as $(1+z)^{2.6}$ [1]). Second, because the UV radiation field is nearly uniform after reionization, it should not imprint any large-scale features on the 21cm power spectrum that would mimic cosmological signatures. On large spatial scales the 21cm sources are expected to have a linear bias analogous to that inferred from galaxy redshift surveys. Since a 21cm survey maps the global hydrogen distribution without resolving individual galaxies, the 21cm bias is expected to be modest compared to surveys that select for the brightest galaxies at the same redshifts.

In general, cosmological surveys are able to measure the power-spectrum of primordial density fluctuations, $P(k)$, to a precision that is ultimately limited by cosmic variance, namely the number of independent Fourier modes that fit within the survey volume. 21cm observations are advantageous relative to existing data sets because they access a 3D volume instead of the 2D surface probed by the cosmic microwave background (CMB), and they extend to a sufficiently high redshift (well beyond the horizon of galaxy redshift surveys [11]) where most of the comoving volume of the observable Universe resides. At these high redshifts, small-scale modes are still in the perturbative (linear growth) regime where their analysis is straightforward. The expected 21cm power extends down to the pressure-dominated (Jeans) scale of the cosmic gas which is orders of magnitude smaller than the comoving scale at which the CMB anisotropies are damped by photon diffusion [3].

Altogether, the above factors make 21cm surveys an ideal cosmological probe of fundamental physics [12]. To illustrate this point, we show in this *Letter* that a dedicated 21cm observatory would enable a determination of the matter power-spectrum at redshifts $z \lesssim 6$ to an unprecedented precision. In our numerical examples, we adopt the standard set of cosmological parameters [13].

Number of Modes. The limitation of existing redshift surveys of galaxies [14, 15] is apparent in Fig. 1 which plots the comoving volume of the Universe out to a redshift z as a function of z. State-of-the-art galaxy redshift surveys, such as the spectroscopic sample of luminous red galaxy (LRGs) in the *Sloan Digital Sky Survey (SDSS)* [11], extend only out to $z \sim 0.5$ (over $\sim 10\%$ of the sky) and probe $\sim 0.01\%$ of the observable Universe.

The CMB fluctuations probe a thin shell on the 2D surface of the sky. The number of modes with a comoving wave number $k \equiv 2\pi/\lambda$ between k and $k+dk$ that fit on this 2D surface is, $dN_{\rm CMB} = \pi k dk \left[\mathcal{A}/(2\pi)^2\right]$, where $\mathcal{A} = D^2 d\Omega$, D is the comoving distance to the last scattering surface at $z \sim 10^3$ and $d\Omega$ is the solid angle of the sur-

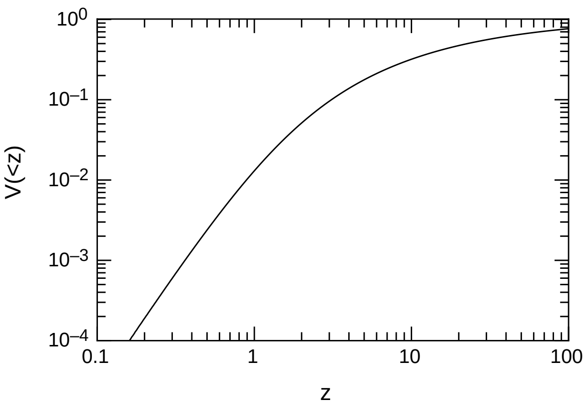

FIG. 1: The fraction of the total comoving volume of the observable Universe that is available up to a redshift z.

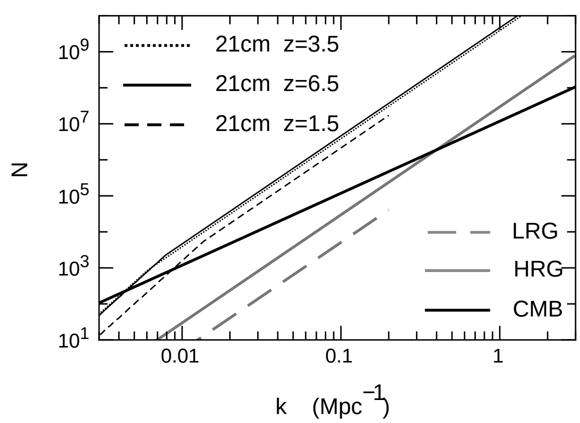

FIG. 2: The number of modes N within a wave number bin of width $\Delta k = k/10$ centered on k, that are available in different cosmological surveys. The thick dashed grey line corresponds to the spectroscopic LRG sample of *SDSS* [11], while the thick solid line (marked HRG) corresponds to a future spectroscopic survey at $2.5 < z < 3.5$ covering 1000 square degrees with a a co-moving galaxy density equal to the LRG sample. The thick dark line corresponds to a CMB data set with $f_{\rm sky} = 0.65$. The thin lines show the number of modes accessible in a 21cm survey (including the limit on large scale modes due to foreground removal [7]) covering $f_{\rm sky} = 0.65$ within a redshift range spanning a factor of 3 in $(1+z)$, and centered on $z = 1.5, 3.5$ and 6.5. For $z \leq 1.5$, we have truncated the curves at $k = 0.2$ Mpc^{-1} to illustrate the smaller range of k accessible within the linear regime at lower redshifts [26].

vey field. Redshift surveys of galaxies and 21cm surveys probe a 3D comoving volume $\mathcal{V}$ and potentially access a larger number of modes, $dN_{\rm 3D} = 2\pi k^2 dk \left[\mathcal{V}/(2\pi)^3\right]$.

Figure 2 compares $N_{\rm CMB} = (k/10)\, dN_{\rm CMB}/dk$ with $N_{\rm 3D} = (k/10)\, dN_{\rm 3D}/dk$, for future 21cm surveys after reionization [5]. The CMB data set is assumed to cover a fraction $f_{\rm sky} = 0.65$ of the sky (excluding the region around Milky Way galaxy). For comparison, we also show the corresponding number of modes within the same k interval in the spectroscopic LRG sample of *SDSS* [11], which covers ~ 3700 square degrees out to $z \sim 0.5$ or a volume of $\mathcal{V} = 0.72h^{-3}{\rm Gpc}^3$ (where $h \approx 0.7$ is the Hubble constant in units of 100km s^{-1} Mpc^{-1}).

21cm observatories that are currently under construction (such as MWA) will survey only a few percent of the sky and process only $\sim 15\%$ of the available frequency range (band-pass). In this *Letter*, we consider future 21cm surveys that would potentially cover $f_{\rm sky} = 0.65$ with a processed frequency-bandwidth spanning a redshift range of a factor of 3 in $(1+z)$ centered on $z = 1.5$, $z = 3.5$ and $z = 6.5$ [37]. With an array design based on MWA in which the effective area of each tile of 16 dipole antennae equals its physical area, the value of $f_{\rm sky} = 0.65$ corresponds to ~ 33 correlated primary beams or fields.

Results. The fractional uncertainties in the 21cm power-spectrum P_{21} for a cosmic-variance limited survey, $(\Delta P_{21}/P_{21}) = 1/\sqrt{N}$, are presented in the inset of Fig. 3 (straight lines). Also shown in both the inset and main panel are the noise curves for observations using a design based on the so-called MWA5000 experiment, which is assumed to have 10 times the collecting area of MWA (as described in Refs. [7, 16]). MWA5000 would be cosmic variance limited in an integration time of $\sim 10^3$ hours at wave numbers near $k \sim 0.1{\rm Mpc}^{-1}$. Since the dipoles of each antenna tile look at $\sim \pi$ steradians, simultaneous processing of multiple primary beams would allow a survey of area $f_{\rm sky} = 0.65$ at 10^3 hours of integration per pointing within a few years. In computing the thermal noise of the observatory, we have adopted a model for the bias factor b_{21} of the 21cm sources ($b_{21}^2 \equiv P_{21}/P$) from Ref. [5]. The noise curves include a limit on large scale modes due to foreground removal. Current estimates for MWA show that foreground removal should be effective for modes over a frequency range $\lesssim$ 6MHz which is $\frac{1}{4}$ of the total 24MHz processed bandwidth [7]. However, improvements on this range would provide access to a larger N as well as to lower-k modes. In Fig. 3 and subsequently, we assume a scenario in which the foreground can be removed on scales of up to $\frac{1}{12}$ of the total processed bandwidth, namely $\frac{1}{12}\times(\sqrt{3}-1/\sqrt{3})[1400{\rm MHz}/(1+z)] = 30[(1+z)/4.5]^{-1}$MHz.

For comparison, we also show the noise curves for the *SDSS*-LRG survey (thick grey line), including the effects of Poisson shot-noise[38], $(\Delta P_{\rm gal}/P_{\rm gal}) = [1 + (b^2 P(k) n_{\rm gal})^{-1}]/\sqrt{N}$, for a galaxy number density of $n_{\rm gal} = 46748/(0.72h^{-3}){\rm Gpc}^{-3}$ and a bias factor of $b_{\rm gal} = 2$ [11]. We find that the potential 21cm constraints on the matter power-spectrum are 1–2 orders of magnitude better than a low-redshift galaxy survey like *SDSS*-LRG.

As an example for the potential use of a 21cm survey, we show in Fig. 3 the expected relative changes in the amplitude of the power-spectrum owing to the presence of a massive neutrino [18]. At wave numbers

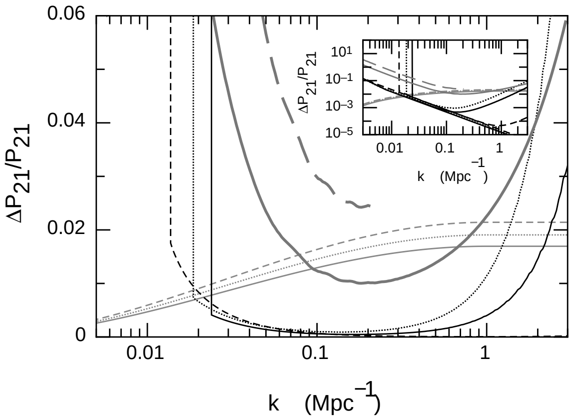

FIG. 3: The fractional change in the amplitude of the power-spectrum owing to the presence of a massive neutrino (horizontal grey lines, asymptoting towards a constant at high k values). The case shown, $f_\nu = 0.004$, corresponds to $m_\nu = 0.05$eV. For comparison, the limits imposed by cosmic variance on measurements of the power-spectrum from the *SDSS*-LRG and a future 1000 square degree galaxy survey at $2.5 < z < 3.5$ are marked by the thick dashed and solid grey lines respectively. The *U*-shaped error curves correspond to an all-sky 21cm survey [$f_{\rm sky} = 0.65$ over a redshift range spanning a factor of 3 in $(1+z)$] with MWA5000 and a 10^3 hour integration per field (line styles for $z = 1.5, 3.5, 6.5$ as in Figure 2). The noise is plotted in bins of size $\Delta k/k = 0.1$. The inset shows these results on logarithmic axes that span a larger dynamic range of achievable precision. The straight thin lines in the inset show the cosmic-variance uncertainty in the power-spectrum measurement owing only to the number of available modes. For $z \le 1.5$, we have truncated the curves at $k = 0.2$ Mpc^{-1} to illustrate the smaller range of k accessible within the linear regime at lower redshifts [26].

much larger than the neutrino free-streaming wave number ($k_{\rm fs} = 0.1\Omega_m(0)h\sqrt{f_\nu}$ Mpc^{-1}), the suppression of the power-spectrum is given by [20, 21],

$$\frac{P(k,f_\nu)}{P(k,f_\nu=0)} = (1-f_\nu)^3\left[1.9\times10^5 g(0)\Omega_m(0)\frac{f_\nu}{N_\nu}\right]^{-6f_\nu/5}, \quad (1)$$

where $g(z) \approx \Omega_m^{0.2}/[1+0.003(\Omega_\Lambda/\Omega_m)^{4/3}]$ is the growth function of the gravitational potential for matter and vacuum density parameters of $\Omega_m(z) = [1+\Omega_\Lambda/\Omega_m(0)(1+z)^3]^{-1}$ and $\Omega_\Lambda = (1-\Omega_m)$ [17]; $f_\nu \equiv [\Omega_\nu(0)/\Omega_m(0)] = 0.08N_\nu(m_\nu/1\text{ eV})$ is the present-day mass fraction of the matter density carried by N_ν neutrino species of particle mass m_ν. In this *Letter* we conservatively assume a non-degenerate hierarchy of neutrino masses with $N_\nu = 1$ and m_ν denoting the largest mass eigenstate. Figure 3 shows the case corresponding to $f_\nu = 0.004$ ($m_\nu = 0.05$eV).

While Fig. 3 indicates that the cosmic variance in a galaxy redshift survey (such as the *SDSS*-LRG survey) is sufficiently small to detect the suppression of power at $k \gg k_{\rm fs}$ due to a neutrino mass of $m_\nu = 0.05$eV, uncertainties in other cosmological parameters and parameter degeneracies reduce this sensitivity by an order of magnitude[18, 19]. Thus, in order to avoid possible systematic offsets between the power-spectrum amplitude observed by different techniques (such as galaxy surveys, CMB maps, and Lyα forest data) at different k values, it is desirable for the 21cm survey to be self-contained and cover a sufficiently broad range of wave numbers that probe the curvature of the neutrino effect in Fig. 3. For this to be achieved, the removal of the Galactic synchrotron foreground would need to be effective over large frequency intervals of up to $\sim 30[(1+z)/4.5]^{-1}$MHz. Foreground removal across such intervals would provide access to the required range of scales over which a constant offset (in the form of a linear bias factor) would not affect the m_ν measurement. The desired wave number to be reached by foreground removal corresponds to the scale where cosmic variance is larger than the change induced by a massive neutrino. Currently, the detailed properties of the foreground are not well measured. The feasibility of foreground removal over a broad frequency interval will remain uncertain until data from the first generation of 21cm observatories is analysed.

Figure 3 illustrates that foreground removal to $\sim 30[(1+z)/4.5]^{-1}$MHz would be sufficient to detect the modification of the power spectrum due to the minimum neutrino mass of 0.047 ± 0.01eV indicated by the latest atmospheric neutrino data [20, 22] at all spatial scales where the effect is larger than cosmic variance. The advantage of the large survey volume is evident since it allows the modification of shape to be measured in addition to the suppression detected by the *SDSS*-LRG survey (the latter being cosmic variance limited on scales where the shape is measured). In a follow-up paper [23], we will address the level of degeneracy with other cosmological parameters. Already, Fisher-matrix studies [7, 24] have demonstrated the improved capabilies of 21cm observations during the epoch of reionization, where contamination from astrophysical sources needs to be removed through the angular dependence of the 21cm power-spectrum [10].

A 21cm survey measures the modulation in the cumulative 21cm emission from a large number of galaxies, as its coarse angular and redshift resolution is not capable of resolving the 21cm sources individually [5, 16]. The damped Lyα systems which contain most of the hydrogen mass in the Universe at $z \lesssim 6$, are expected to be hosted by abundant low mass galaxies [25] and thus have a weak bias relative to the underlying matter distribution on large spatial scales. This weak bias is not expected to introduce a feature to the power-spectrum that is degenerate with the neutrino signature (as would be the case prior to reionization). Although the comoving wave number at which non-linear evolution becomes important increases from $k \sim 0.1h^{-1}$Mpc^{-1} at $z = 0.3$ to $\sim 0.5h^{-1}$Mpc^{-1} at $z = 3$ [26], the constraints on m_ν

can be potentially improved by accounting for the related non-linear effects [28].

By measuring the evolution of the growth factor with redshift to the exquisite precision implied by Fig. 3, it would also be possible to constrain alternative theories of gravity or dark energy well beyond the capabilities of existing data sets [29]. The evolution of the growth factor would be limited by the knowledge of the bias of the 21cm sources, b_{21}. The limit on the uncertainty in growth factor would satisfy $[d\ln g/d\ln(1+z)] \approx [d\ln b_{21}/d\ln(1+z)]$.

Finally, we note that a precise $P(k)$ measurement at multiple redshifts would also allow to determine the redshift evolution of the baryonic acoustic oscillations (BAO) in the 21cm power spectrum [5, 30]. The BAO scale constitutes a standard ruler [26, 27] that can be used to measure the equation of state of the dark energy [6, 16], constrain $1-\Omega_{\rm tot}$, and further remove degeneracies between m_ν and other cosmological parameters [31].

Hardware. The required 21cm observatory could be similar in antenna design to the planned MWA but would require expanded collecting area and computer resources to account for the increased cross-correlation requirements and the analysis of multiple beams. A suitable observatory would contain ten times the number of tiles in MWA. The computational load increases as the square of the number of tiles in a telescope, and linearly with both the amount of processed band-pass and the number of cross-correlated primary beams. Thus, an all-sky survey over a frequency range covering a factor of 3 in $(1+z)$ requires an overall improvement by $\sim 10^4$ in computer power relative to MWA[39].

Acknowledgments. We thank M. McQuinn and M. Zaldarriaga for helpful comments. We also thank the Harvard-Australia foundation for funding a visit of A.L. to Australia, during which this work was performed.

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[37] The factor of 3 in $(1+z)$ corresponds to the largest frequency bandwidth over which a low-frequency dipole antenna has suitable sensitivity.
[38] We ignore Poisson fluctuations in the 21cm power spectra since the contributing galaxies are expected to be of much lower mass than LRGs, and there should be a large number of them per resolution element of the survey.
[39] The cost of the antenna hardware is expected to be in the range of hundreds of million of dollars, an order of magnitude higher than for MWA. Improved telescope designs [24] and advances in computer technology could help to realize our projected 21cm observatory.